Exercises in relativity

Einstein's special theory of relativity predicts some strange effects as an object approaches the speed of light. According to the theory:

- Nothing can travel faster than the speed of light.
- Objects with no mass (eg photons) can travel at the speed of light.
- Objects with mass can't travel at the speed of light.

This worksheet will help you to understand why real objects can't travel at the speed of light.

When answering the following questions, use as many significant figures as your calculator can show.

The effect of relative motion on mass

- The special theory of relativity predicts that an object's mass will change as its speed increases. The relativistic mass m_{rel} of an object travelling at speed v is given by:

$$m_{rel} = \frac{m}{\sqrt{1-v^2/c^2}}$$

where m is the rest mass of the object and c is the speed of light (2.99792458×10^8 m s^{-1}).

1. Use the formula to calculate the relativistic mass of a 100 kg person travelling at 1000 m s^{-1}.

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2. Calculate the relativistic mass of the person travelling at 50% of the speed of light.

- The Australian synchrotron uses electric and magnetic fields to accelerate electrons around a circular track. As they travel around the track, their acceleration causes them to emit electromagnetic waves (light). This light is used in research programs to investigate the structure of matter.

3. Use the formula to calculate the relativistic mass of an electron travelling around the synchrotron at 99% of the speed of light. (Rest mass of an electron is $9.1093897 \times 10^{-31}$ kg).

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4. Why is more energy needed to accelerate an electron as its speed increases?

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5. Is it possible for a synchrotron to accelerate electrons to the speed of light? Explain your answer.

The effect of relative motion on length

- Special relativity predicts what happens when two observers measure the length of an object, where one observer is at rest relative to the object, and the other observer is moving relative to the object. The length of the object in the direction of travel, as measured by observer who is moving relative to the object, is smaller than the length measured by the observer who is at rest relative to the object. The formula for the relativistic length L' of an object travelling at speed *v* relative to an observer is:

  $$L' = L\sqrt{1 - v^2/c^2}$$

  where *L* is the length of the object measured by an observer at rest relative to the object.

6. Use the formula to calculate the relativistic length of a 100 m long spaceship travelling at 3000 m s^{-1}.

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7. Calculate the relativistic length of the spaceship if it travels at 80% of the speed of light.

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8. Use the formula to explain why it is impossible for a spaceship to travel at the speed of light.

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The effect of relative motion on time

Muons are produced at the outer edge of our atmosphere when cosmic radiation interacts with air. They are unstable particles with a half-life of 1.5 x 10^{-6} seconds. Although they rain down on us at almost the speed of light, most muons decay before reaching Earth's surface. In one half-life, muons should travel about 450 m, by which time 50% will have decayed.

In 1941, a detector located near the top of Mt Washington (1830 m above sea level) recorded 570 muons per hour. When the detector was placed at sea level, it recorded 400 muons per hour; not the 35 predicted from the decay rate. Scientists were puzzled to explain why the muons survived so long.

The explanation is found in the special theory of relativity. Because muons travel at almost the speed of light, time dilation and length contraction effects are significant.

An observer on Mt Washington explains the discrepancy by the longer lifetime of the muons — 'time runs slowly for the moving muons'. An observer moving with the muons explains the discrepancy by length contraction of the space between the top of the mountain and sea level, which they see rushing towards them. From the muons' reference frame, the top of Mt Washington is only 200 m above sea level — one ninth of its height as measured by an observer at rest relative to the mountain height. This explains why muons reach sea level so quickly and why many more of them survive.

Reference:
Fowler, M. (n.d.). Experimental Evidence for Time Dilation: Dying Muons. Retrieved 20 Oct 2009 from http://galileoandeinstein.physics.virginia.edu/lectures/srelwhat.html

9. Use the information above to explain why scientists expected to detect only 35 muons per hour at sea level.

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10. Use the formula for relativistic length to show that, for a muon travelling at 99.4% of the speed of light, the distance from the top of Mt Washington to sea level is only 200 m.

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- Special relativity also predicts that two observers moving relative to each other will each see that the other's clock ticks at a different rate. The time difference depends on the relative speeds of the observers.

For an observer travelling at speed *v* relative to a clock, the time interval *Δt′* that they measure for an elapsed time *Δt* on the clock is:

Each observer will see that time varies for the other person by a factor γ, where

11. Calculate the value of γ for the time difference between two clocks: one at rest relative to you, and one in a car travelling at 60 km h^{-1} relative to you (16.67 m s^{-1}). Comment on the time difference.

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12. Calculate the value of γ for the time difference between a stationary clock on Earth, and one in a GPS satellite orbiting the Earth at 3.87 x 10^3 m s^{-1}. Comment on your answer.

- Finally, a question to think about and discuss with other students.

13. In the Large Hadron Collider, a beam of protons is accelerated to 99% of the speed of light before smashing into a similar beam travelling in the opposite direction. Estimate the speed of impact between colliding protons.

Notes

1. A clock on an orbiting satellite may run faster or slower than a clock on Earth because:
   a. the satellite's speed makes its clock run slower (special theory of relativity), and
   b. the weaker gravity at the satellite's altitude makes its clock run faster (general theory of relativity).
2. A stationary observer, watching an object travelling at almost the speed of light, would see that its mass, length and rate of flow of time have all changed. However, an observer travelling at the same speed as the object wouldn't see any change in its mass, length or time.
3. But, because all motion is relative, the observer moving with the object at almost the speed of light, would see that mass, length and rate of flow of time for the stationary observer have all changed.