The University of Western Australia
DEPARTMENT OF MATHEMATICS AND STATISTICS

BLAKERS MATHEMATICS COMPETITION

2021 Problems

The Competition begins Friday, 28 May and ends Friday, 13 August 2021.

You may use any source of information except other people. In particular, you may use computer packages (though contributed solutions are expected to have complete mathematical proofs). Extensions and generalisations of any problem are invited and are taken into account when assessing solutions. Also, elegance of solution is particularly favoured and desired. We reserve the right to not read a solution if it is "messy"; we expect neat solutions (so perhaps avoid submitting a first draft).

Solutions are to be emailed as PDF attachments to Greg Gamble (`greg.gamble@uwa.edu.au`) **by 4 pm on Friday, 13 August.**

Remember, you don't have to solve all the problems to win prizes!

Instructions for solutions: Include the following information in the body of your email:

name,
student ID number,
home address (optional),
e-mail address,
university where enrolled,
number of years you have been attending any tertiary institution, and
list of the questions completed and attached as PDFs to your email.

Please scan each question to a separate PDF file and name the file according to the protocol: ⟨`YourLastName`⟩⟨n⟩`.pdf` where ⟨n⟩ is the number of the question.
Before you scan, please ensure you have started each new problem on a new page, and that at the top of each page is the number of the page, and your full name.

Note. Our convention is that $\mathbb{N} = \{1, 2, \dots\}$ (the positive integers).

1. Truth in inequality

Prove that

$$x^2 + y^2 + 2 \geqslant (x + 1)(y + 1),$$

for all $x, y \in \mathbb{R}$. When does equality hold?

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2. Naturally cyclic

Find all positive integer quadruples (a, b, c, d) satisfying

$$ab + bc + cd + da = 2021.$$

How many such solutions are there?

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3. Triply real

Find all real triples (x, y, z) that satisfy the equations

$$x + \frac{1}{x} = 2y^2 \tag{1}$$

$$y + \frac{1}{y} = 2z^2 \tag{2}$$

$$z + \frac{1}{z} = 2x^2 \tag{3}$$

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4. Dinothesaurus

A mad editor has undertaken to publish the list, in alphabetical order, of all the "words" of 26 letters comprising each of the letters of the roman alphabet, exactly once. The gigantic list is to appear in 21 volumes each containing the same number of words. So the first word of the first volume will be

$$abcdefghijklmnopqrstuvwxyz$$

followed by

$$abcdefghijklmnopqrstuvwxzy.$$

What will be the last word of the first volume?

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5. An incentred triangle

Let triangle ABC have incentre I. Let A', B', C' be the reflections of I in sides BC, CA, AB, respectively.

Prove that $\angle A'B'C'$ does not depend on $\angle BAC$, and find $\angle A'B'C'$ in terms of $\angle ABC$.
